TALK INFORMATION

1. Mini-courses

(1) **Speaker:** Konstanze Rietsch (King's College London)

Title: Positivity and mirror symmetry for homogeneous spaces.

Abstract: Projective homogeneous spaces G/P such as Grassmannians and flag varieties have rich structures coming from their symmetry groups and their relationship with representation theory. For example, Lusztig's theory of 'total positivity' gives rise to a 'positive' subset of G/P, with this positivity related to positivity properties of canonical bases. Also, Langlands duality of reductive algebraic groups induces a duality of homogeneous spaces. In these lectures I will talk about mirror symmetry in the context of the homogeneous spaces G/P, and show how mirror symmetry interacts with structures from Lie theory/representation theory in these examples.

(2) **Speaker:** Richard Rimanyi (University of North Carolina at Chapel Hill)

Title: Characteristic classes of singular spaces in cohomology, K-theory, and elliptic cohomology.

Abstract: Recent works of Maulik-Okounkov, Gorbounov-Rimanyi-Tarasov-Varchenko and others identify two modules: the Bethe algebra of certain quantum integrable systems, and the regular representation of the equivariant cohomology (K-theory, elliptic cohomology) algebra of certain homogeneous spaces. A remarkable feature of this identification is that 'easy objects on one side correspond to 'difficult objects on the other side. We will survey the key points of this identification through small examples. In cohomology we obtain effective algorithms and formulas for the MacPherson deformation of Schubert classes, Thom polynomials, and quiver polynomials. In K-theory we present analogous results on the Hirzebruch classes.

2. Conference Talks

(1) **Speaker:** Hiraku Abe (Osaka City University Advanced Mathematical Institute)

Title: Cohomology of Hessenberg varieties and representations of symmetric groups.

Abstract: Hessenberg varieties are subvarieties of the full flag variety, and it is a relatively new research subject which was introduced by De Mari-Procesi-Shayman around 1990 based on a connection to QR-algorithm for matrices. Particular examples of Hessenberg varieties are

- Springer fibres (from geometric representation theory),
- Peterson variety (related to Quantum cohomology of partial flag varieties),
- maximal torus orbit closures (a toric variety arising from Lie theory).

Similarly to Schubert varieties, it has been found that geometry, combinatorics, and representation theory interact nicely on Hessenberg varieties as well, and yet there are

still many things to be studied. In this talk, after a brief introduction of an example of this interaction, I will explain the relation between the cohomology rings of regular nilpotent Hessenberg varieties and the regular semisimple Hessenberg varieties in terms of representations of the symmetric group, in Lie type A. This is joint work with Megumi Harada, Tatsuya Horiguchi, and Mikiya Masuda.

(2) **Speaker:** David E. Anderson (Ohio State University)

Title: New formulas for degeneracy loci

Abstract: Over the last 25 years, many extensions of Giambelli's determinantal formula have been found (many of them by participants in this conference). These include versions related to classical groups, as well as formulas valid in K-theory or other generalized cohomology theories. In this talk, I will describe some of these formulas, focusing especially on determinants and Pfaffians in K-theory. I will also mention an application toward computing the genus of a Brill-Noether variety parametrizing special divisors on a curve. Much of this is joint work with William Fulton, Linda Chen and Nicola Tarasca.

(3) **Speaker:** Sara C.Billey (University of Washington)

Title: Applications of Transition Equations to Macdonald's Formula and Beyond

Abstract: In 1982, Lascoux and Schutzenberger introduced the Transition Equation for Schubert polynomials. This recurrence relation has been used and generalized in many ways related to Stanley symmetric functions and Schubert polynomials in other types. We will review some of the applications and generalizations in this talk. One recent application is a new bijective proof of Macdonald's Reduced Word Formula using pipe dreams and Little's bumping algorithm. This remarkable formula equates a weighted sum of reduced words for a permutation with the number of terms in a Schubert polynomial. This proof extends to a principal specialization due to Fomin and Stanley using a new variation on the Transition Equation. This approach has been sought for over 20 years. Our bijective tools also allow us to address a problem posed by Fomin and Kirillov from 1997 using work of Wachs, Lenart, Serrano and Stump. This project extends earlier work by the third author on a Markov process for reduced words of the longest permutation.

This talk is based on joint work with Ben Young and Alexander Holroyd.

(4) **Speaker:** Anders S. Buch (Rutgers University)

Title: Puzzles for projections from 3-step flag varieties

Abstract: I will speak about a positive formula for the cohomological Schubert structure constants of a 3-step flag variety, when one of the involved Schubert classes is pulled back from a 2-step variety or a Grassmannian. This can be interpreted either as a formula for special products on the 3-step flag variety, or as a formula for the projection of an arbitrary product of Schubert classes. The formula counts puzzles made from puzzle pieces that are constructed in a systematic way. In addition, I will explain a combinatorial Borel construction that generates equivariant puzzle pieces from ordinary pieces. This construction also recovers pipe dreams as the puzzles associated to projections to a point. The formulas are proved using my mutation algorithm for puzzles.

As an application, I obtain a new Littlewood-Richardson rule for the (equivariant, 3 point, genus zero) Gromov-Witten invariants of Grassmannians that does not produce any irrelevant puzzles.

(5) **Speaker:** Baptiste Calmès (Université d′Artois)

Title: Schubert Calculus for oriented cohomology theories.

Abstract: I'll survey a generalization of classical Schubert calculus to any given oriented cohomology theory, contained in several papers by Petrov, Zainoulline, Zhong and myself. It takes as input the formal group law of the oriented cohomology theory, the cohomology of the point and root datum combinatorics corresponding to a flag variety of any type, and outputs an algebraic description of the oriented cohomology of the flag variety (or a torus-equivariant version). One recovers classical Schubert calculus - Chow groups or K-theory - using the additive or multiplicative group laws. The universal case is algebraic cobordism.

(6) **Speaker:** Haibao Duan (Chinese Academy of Sciences)

Title: Schubert claculus and Chomology of Lie groups

Abstract: The problem of computing the cohomologies of Lie groups was raised by E. Cartan in 1929, which has been a focus of algebraic topology for the fundamental roles of Lie groups playing in geometry and topology. On the other hand Schubert calculus began with the intersection theory of the 19th century. Clarifying this calculus has been a major theme of algebraic geometry. Let G be a compact Lie group with a maximal torus T. In this talk I will show how Schubert calculus on G/T could be applied to give a unified construction of the integral cohomology of the Lie group G.

(7) **Speaker:** Vassily Gorbounov (University of Aberdeen)

Title: Schubert calculus and quantum integrable systems

Abstract: We will show how the quantum equivariant Schubert calculus is related to an interesting quantum integrable sysytem or a quantum group in the terminology of the L.D. Faddeev school. We outline some applications of this connection. We will show how our quantum group is related to well studied quantum groups, namely to the Yangian of $SL(2)$.

(8) **Speaker:** Tatsuya Horiguchi (Osaka University / Osaka City University Advanced Mathematical Institute)

Title: Hessenberg varieties and hyperplane arrangements

Abstract: Hessenberg varieties are subvarieties of the full flag variety. The study of the topology of Hessenberg varieties makes connection with many research areas such as: geometric representation theory, quantum cohomology of flag varieties, and graph theory. Recently, Sommers and Tymoczko conjectured that the Poincare polynomial of arbitrary regular nilpotent Hessenberg variety coincides with some polynomial defined by certain hyperplane arrangement. In this talk, I will introduce a ring whose Hilbert series is the polynomial defined by the hyperplane arrangement above, and I will explain

that this ring is isomorphic to the cohomology ring of the regular nilpotent Hessenberg variety. In particular, we will see that Sommers-Tymoczko conjecture is true. If time permits, I will explain the application of this result. This is joint work with Takuro Abe, Mikiya Masuda, Satoshi Murai, and Takashi Sato.

(9) **Speaker:** Takeshi Ikeda (Okayama University of Science)

Title: A K-Peterson isomorphism from integrable systems

Abstract: I will discuss a candidate for the K-theoretic Peterson isomorphism from a view point of integrable systems. We give an explicit isomorphism between the K-homology ring of the affine Grassmannian of type A, and a conjectural form of the quantum K-ring of the flag variety, with both rings appropriately localized. The map comes from a special solution of the relativistic Toda lattice due to Ruijsenaars. The solution is the one whose Lax matrix is unipotent, and it is written in terms of the dual stable Grothendieck polynomials. The talk is based on a joint work with S. Iwao and T. Maeno.

(10) **Speaker:** Bumsig Kim (Korea Institute for Advanced Study)

Title: Virtual cycles via two-periodic localized Chern characters

Abstract: The localized Chern character of a bounded complex of vector bundles is a bivariant class defined by Baum, Fulton, and MacPherson. They used such classes to prove a general Riemann- Roch theorem for singular varieties. For a two-periodic complex of vector bundles, Polishchuk and Vaintrob have constructed its localized Chern character, which is a generalization of the usual one. We discuss some properties of PVs localized Chern characters. In particular, cosection localizations defined by Kiem and Li can be expressed as these localized Chern character operations. This result is a generalization of the related work by Chang, Li, and Li. The talk is based on joint work with Jeongseok Oh.

(11) **Speaker:** Valentina Kiritchenko (National Research University Higher School of Economics)

Title: Schubert calculus on convex polytopes

Abstract: I will talk about a convex geometric approach to Schubert calculus. Using ideas from the theory of Newton polytopes we can represent Schubert cycles by linear combinations of faces of polytopes from representation theory. The intersection product of cycles corresponds to the set-theoretic intersection of faces (whenever the latter are transverse). There is an algorithm (geometric mitosis) for finding positive presentations of Schubert cycles by faces. For Gelfand-Zetlin polytopes in type A, this algorithm reduces to Knutson-Miller mitosis on pipe dreams. In types B and C, geometric mitosis reduces to a different combinatorial rule that conjecturally yields presentations of Schubert cycles by faces of symplectic Gelfand-Zetlin polytopes.

(12) **Speaker:** Allen Knutson (Cornell University)

Title:

Abstract:

(13) **Speaker:** Seung Jin Lee (Seoul National University)

Title: Affine Schubert polynomials

Abstract: In this talk, we introduce affine Schubert polynomials that represent cohomology classes of Schubert cycles in affine flag variety for type A. It turns out that these polynomials both generalize affine Stanley symmetric functions and Schubert polynomials. We also present a coproduct formula of the affine Schubert polynomial which recovers the Schubert polynomials formula by Billey and Haiman. If time permits, we also discuss a monomial expansion of affine Schubert polynomials and its equivariant version.

(14) **Speaker:** Tomoo Matsumura (Okayama University of Science)

Title: Determinant formulas in Schubert calculus

Abstract: The double Schubert polynomials introduced by Lascoux and Schützenberger coincide with the Schubert classes in the equivariant cohomology of complete flag varieties of type A, when the variables are specialized to roots of tautological bundles. Those which correspond to 2143 avoiding and 321 avoiding permutations are known to have a determinant formula and a tableau formula. I will report on the recent progress generalizing these facts to K-theory, namely, a determinant formula and a tableaux formula of the double Grothendieck polynomials associated to these two families of permutations.

(15) **Speaker:** Leonardo C. Mihalcea (Virginia Tech)

Title: Chern-Schwartz-MacPherson classes for Schubert cells, characteristic cycles, and positivity

Abstract: A compact manifold has a tangent bundle, and a natural question is to find a replacement for the total Chern class of the tangent bundle, in the case when the space is singular. The Chern-Schwartz-MacPherson (CSM) classes are homology classes which 'behave like' the total Chern class of the tangent bundle, and are determined by a functoriality property. The existence of these classes was conjectured by Grothendieck and Deligne, and it was proved by MacPherson in 1974. The calculation of the CSM classes for Schubert cells and Schubert varieties in flag manifolds was obtained only recently, and it exhibited some interesting features. For instance, the classes of Schubert cells are determined by certain Demazure-Lusztig operators. They are essentially equivalent to the characteristic cycles of the Verma modules in the cotangent bundle of the complete flag manifold, and to the stable envelopes of Maulik and Okounkov. We used all of this to find their Poincaré duals, and to prove that they are Schubert positive. I will survey these developments. This is based on joint work with P. Aluffi, J. Schürmann and C. Su.

(16) **Speaker:** Oliver Pechenik (University of Michigan)

Title: Decompositions of Grothendieck polynomials

Abstract: Finding a combinatorial rule for the Schubert structure constants in the K-theory of flag varieties is a long-standing problem. The Grothendieck polynomials of

Lascoux and Schützenberger (1982) serve as polynomial representatives for K-theoretic Schubert classes in type A, but no positive rule for their multiplication is known in general. We contribute a new basis for polynomials (in n variables), give a positive combinatorial formula for the expansion of Grothendieck polynomials in these "glide polynomials," and provide a positive combinatorial Littlewood-Richardson rule for expanding a product of Grothendieck polynomials in the glide basis. (Joint work with Dominic Searles.)

(17) **Speaker:** Nicolas Perrin (Université de Versailles)

Title: Geomety of horospherical varieties of Picard rank one

Abstract: Pasquier classified all horospherical varieties of Picard rank one. These are examples of Fano varieties with a big automorphism group but which are not homogeneous. We will describe their common geometry with a view towards rational curves and quantum cohomology. Joint work with R. Gonzales, C. Pech and A. Samokhine.

(18) **Speaker:** Piotr Pragacz (Polish Academy of Sciences)

Title: Gysin formulas for flag bundles

Abstract: We give push-forward formulas for all flag bundles of types A, B, C, D. The formulas (and also the proofs) involve only Segre classes of the original vector bundles and characteristic classes of universal bundles. We also establish a Gysin formula for Kempf-Laksov bundles and their isotropic analogues. This is a joint work with Lionel Darondeau.

(19) **Speaker:** Arun Ram (University of Melbourne)

Title: Combinatorics of semi-infinite flag varieties

Abstract: I will discuss the path model/alcove walk model for the semi-infinite flag variety. This model has connections to the path model formula for Macdonald polynomials and to the Littelmann path model for representations of affine Lie algebras at level 0.

(20) **Speaker:** Vijay Ravikumar (Chennai Mathematical Institute)

Title: Equivariant Pieri Rules for Isotropic Grassmannians

Abstract: We describe a manifestly positive Pieri rule for the torus-equivariant cohomology of Grassmannians of Lie types B, C, and D. To the best of our knowledge, this is the first such formula for sub-maximal Grassmannians. Our method involves reducing equivariant Pieri coefficients to restrictions of special Schubert classes at torus fixed points in the equivariant cohomology ring of a different Grassmannian. This is joint work with Changzheng Li.

(21) **Speaker:** Mark Shimozono (Virginia Tech)

Title: Formal back-symmetric power series and Schubert calculus of infinite flags

Abstract: We describe an explicit ring of formal power series which realizes the cohomology of infinite flags, including explicit divided difference operators. The Schubert basis is given by Allen Knutson's back-stable Schubert polynomials. We give a coproduct formula for them in terms of Stanley symmetric functions and ordinary Schubert polynomials. I believe this is a new formula even in the context of the ordinary flag variety and Grassmannian. These also have equivariant analogues; the equivariant Stanley functions are new. We also make some conjectures regarding the equivariant cohomology ring of the "affinization" of the Cartan data of the infinite flag ind-variety, with respect to loop rotation and the usual torus. Joint work with Thomas Lam and Seung Jin Lee.

(22) **Speaker:** Changjian Su (IHES)

Title: Macdonald and Casselman–Shalika formula from Schubert calculus

Abstract: In this talk, I will compare two geometric realization of the periodic modules for the affine Hecke algebra. One is using equivariant K-theory of T^*G/B, while the other one involves unramified principle series of the p-adic Langlands dual group. We compare two bases between them. With this, we can have an equivariant K-theoretic interpretation of the Macdonald's formula for the zonal spherical functions and Casselman–Shalika formula for the Whittaker functions. This is my joint work with Changlong Zhong and Gufang Zhao.

(23) **Speaker:** Valentin Tonita (MPIM Bonn)

Title: A reconstruction theorem in quantum K-theory

Abstract: I will explain the D-module structure in quantum K-theory (joint work Givental,T.): it can be thought of as a replacement of the divisor equation. I will then show how this leads to a reconstruction of the quantum K-theoretic product from the small J-function (joint work Iritani, Milanov, T.), emphasizing (potential) applications to quantum K-theoretic Schubert calculus.

(24) **Speaker:** Andrzej Weber (University of Warsaw)

Title: Hirzebruch classes of matrix Schubert varieties as envelopes

Abstract: In their recent preprint Feher and Rimanyi have shown that the equivariant Chern-Schwartz-MacPherson classes of matrix Schubert varieties satisfy certain axioms allowing to apply the restriction method in computations. As a consequence, CSM classes coincide with the weight functions defined by Rimanyi-Tarasov-Varchenko. We will prove analogous set of axioms for the Hirzebruch class, or equivalently for the K-theoretic analogue of the CSM-class. This way we prove that Hirzebruch classes of matrix Schubert varieties constitute stable K-theoretic envelope. The proof is based on the asymptotic properties of the equivariant Hirzebruch class and its relation with Bialynicki-Birula decomposition. The results hold in a greater generality, namely for an algebraic group acting on a vector space with finitely many orbits. Joint work with

L. Feher and R. Rimanyi.

(25) **Speaker:** Changlong Zhong (SUNY-Albany)

Title: Hecke algebra, stable bases and hyperbolic Schubert calculus.

Abstract: In this talk, I will talk about two applications of Hecke algebras. The first one is about K-theoretic stable bases. By using the twisted group algebra of Kostant and Kumar, I will give an algebraic definition of K-theoretic stable basis originally defined by Maulik and Okounkov. Then I will talk about the formal root polynomial method, which is used to compute the restriction formula of stable basis. This is joint work with Changjian Su and Gufang Zhao. The second application is done jointly with Cristian Lenart and Kirill Zainoulline. It is about some generalized oriented cohomology theory (called hyperbolic cohomology) of flag varieties. The Kazhdan-Lusztig basis of Hecke algebra will be used to study smoothness of Schubert varieties.