Quasi-elliptic cohomology

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Overview

Plan.

- The construction of quasi-elliptic cohomology
- The power operation
- The orthogonal $G-$spectra

Motivation

An old idea of Witten [Landweber]

The elliptic cohomology of a space X is related to the $\mathbb{T}-$equivariant K-theory of $LX = \mathbb{C}^\infty(S^1, X)$ with the circle $\mathbb{T}$ acting on LX by rotating loops.

It's surprisingly difficult to make this precise.

Why?

In application, one needs to consider the case that a group G acts on X. In this case the loop space LX has rich structures as an orbifold.

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Bibundles $\sim$ "bimodules" in geometry

Bibundles combine several widely used notions, including smooth maps, Lie homomorphisms, and principal bundles.

A bibundle from $\mathbb{H}$ to $\mathbb{G}$ [Schommer-Pries] [Lerman]

a smooth manifold P together with

- the structure maps:
  - $\tau : P \longrightarrow \mathbb{G}_0$;
  - a surjective submersion $\sigma : P \longrightarrow \mathbb{H}_0$.
- The action maps in $Man_{G_0 \times H_0}$
  - $\mathbb{G}_1 \,{}_s\!\times_\tau P \longrightarrow P$;
  - $P \,{}_\sigma\!\times_t \mathbb{H}_1 \longrightarrow P$

such that

- $g_1 \cdot (g_2 \cdot p) = (g_1 g_2) \cdot p$; • $(p \cdot h_1) \cdot h_2 = p \cdot (h_1 h_2)$; • $g \cdot (p \cdot h) = (g \cdot p) \cdot h$
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Example ($Loop(X//G) := Bibun(S^1//*, X//G)$)

- Objects:

$$\mathcal{P} := \{S^1 \xleftarrow{\pi} P \xrightarrow{f} X\}$$

- Morphisms:

$$\begin{tikzcd} S^1 & P \arrow[l, "\pi"'] \arrow[r, "f"] \arrow[d, "\alpha"] & X \\ & P' \arrow[ul, "\pi'"] \arrow[ur, "f'"'] & \end{tikzcd}$$

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The isotropy groups in $Loop^{ext}(X//G)$ may be infinite dimensional topological groups when G is not finite.

the subgroupoid $\Lambda(X//G)$ instead

$$\Lambda(X//G) := \coprod_{g \in G^{tors}_{conj}} X^g // \Lambda_G(g)$$

G^{tors}_{conj}: a set of representatives of $G-$conjugacy classes in G^{tors};

$$\Lambda_G(g) = C_G(g) \times \mathbb{R}/\langle (g, -1) \rangle$$

$QEll$ as equivariant $K-$theories

$$QEll_G(X) \cong \prod_{g \in G^{tors}_{conj}} K_{\Lambda_G(g)}(X^g)$$

Relation with Tate K-theory

$$QEll^*_G(X) \otimes_{\mathbb{Z}[q^{\pm}]} \mathbb{Z}((q)) \cong K^*_{Tate}(X//G).$$

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Quasi-elliptic cohomology has power operations, which gives it the structure of an "$H_\infty-$ring theory" [Ganter 06].

Atiyah's Power Operation [Ganter]

V: a vector bundle over $\Lambda(X//G)$.

$P_n(V) := V^{\widehat{\otimes}_{\mathbb{Z}[q^\pm]} n}$ defines an operation

$$P_n : QEll_G(X) \longrightarrow QEll_{G\wr\Sigma_n}(X^{\times n})$$

$$\mathbb{P}_n = \prod_{(\underline{g},\sigma)\in(G\wr\Sigma_n)^{tors}_{conj}} \mathbb{P}_{(\underline{g},\sigma)} :$$

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Finite Subgroups of Tate Curve

Theorem (Huan)

$$QEll(pt//\Sigma_N)/\mathcal{I}_{tr}^{QEll} \cong \prod_{N=de} \mathbb{Z}[q^{\pm}][q'^{\pm}]/\langle q^d - q'^e \rangle,$$

where $\mathcal{I}_{tr}^{QEll}$ *is the transfer ideal and* q' *is the image of* q *under the power operation* $\mathbb{P}_N$*. The product goes over all the ordered pairs of positive integers* (d, e) *such that* $N = de$*.*

Theorem (Huan)

The Tate K-theory of symmetric groups modulo a certain transfer ideal classifies finite subgroups of the Tate curve.

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Orthogonal $G-$Spectra

Goerss-Hopkins-Miller theorem constructs many example of $E_\infty-$rings which represent elliptic cohomology theories, including Tate K-theory.

Question

Can we construct $E_\infty - G-$Spectrum which represents equivariant elliptic cohomology theory (e.g. $G-$equivariant Tate K-theory)?

Orthogonal $G-$spectra of Quasi-elliptic cohomology [Huan]

We construct a commutative $\mathcal{I}_G-$FSP $(E(G,-),\eta,\mu)$. For each faithful $G-$representation V, $E(G,V)$ weakly represents $QEll_G^V(-)$ in the sense

$$\pi_k(E(G,V)) = QEll_G^V(S^k), \text{ for each } k.$$

Can $E(G,-)$ arise from an orthogonal spectrum?

No.

For a trivial $G-$representation V, the $G-$action on $E(G,V)$ is not trivial.

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a new Global Homotopy Theory

Global Homotopy Theory [Schwede][May]

Observation: It has been noticed since the beginnings of equivariant homotopy theory that certain theories naturally exist not just for a particular group, but in a uniform way for all groups in a specific class.

$\Rightarrow$ **global homotopy theory**

Prominent examples: equivariant stable homotopy, equivariant K-theory, equivariant bordism.

Almost Global Homotopy Theory [Huan]

- an extension of global homotopy theory;
- classifies those theories that are almost "global";
- the restriction maps are equivariant weak equivalence.

We can define global quasi-elliptic cohomology. [Huan]

Combining the orthogonal G−spectra $\{E(G,-)\}$, we get an ultra-commutative global ring spectrum in the new theory.

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Observation: It has been noticed since the beginnings of equivariant homotopy theory that certain theories naturally exist not just for a particular group, but in a uniform way for all groups in a specific class.
$\Rightarrow$ **global homotopy theory**
Prominent examples: equivariant stable homotopy, equivariant K-theory, equivariant bordism.

Almost Global Homotopy Theory [Huan]

- an extension of global homotopy theory;
- classifies those theories that are almost "global";
- the restriction maps are equivariant weak equivalence.

We can define global quasi-elliptic cohomology. [Huan]

Combining the orthogonal $G-$spectra $\{E(G,-)\}$, we get an ultra-commutative global ring spectrum in the new theory.

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Model Structure on the almost global spaces

We formulate several model structures and are formulating the one below.

Conjecture

There is a global model structure on the almost global spaces that is Quillen equivalent to the global model structure on the orthogonal spaces formulated by Schwede in Global Homotopy Theory.

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Thank you.

Some references

http://www.math.uiuc.edu/~huan2/Zhen-Regensburg-2017-Slides.pdf

- Ando, "Isogenies of formal group laws and power operations in the cohomology theories E_n", Duke J., 1995
- Ando, Hopkins, Strickland, "The sigma orientation is an H_∞ map", Amer. J. 2004; arXiv:math/0204053
- Adem, Leida, Ruan, "Orbifolds and stringy topology", Cambridge Tracts in Mathematics, 171. 2007.
- Atiyah, "K-theory", second edition, 1989. Notes by D.W.Anderson.
- Atiyah, "Power operations in K-theory", Quart. J. Math. Oxford Ser. (2) 17 1966.
- Atiyah,"Equivariant K-theory and completion", J. Differential Geometry 3 1969.
- Berger, Moerdijk, "On an extension of the notion of Reedy category", Mathematische Zeitschrift, December 2011.
- Bröcker, Dieck, "Representation of Compact Lie Groups", Springer GTM **98** 1985.
- Bruner, May, McClure, Steinberger, "H_∞ ring spectra and their applications", volume 1176 of Lecture Notes in Mathematics, 1986.
- Ganter, "Orbifold genera, product formulas, and power operations", Adv. Math, 2006; arXiv:math/0407021.
- Ganter, "Stringy power operations in Tate K-theory", Homology, Homotopy, Appl., 2013; arXiv:math/0701565
- Ganter, "Power operations in orbifold Tate K-theory"; arXiv:1301.2754
- Hirschhorn, "Model categories and their localizations", Mathematical Surveys and Monographs, 99. American Mathematical Society, 2003.
- Hopkins, Kuhn, Ravenel, "Generalized group characters and complex oriented cohomology theories", J. Am. Math. Soc. 13 (2000).
- Huan, "Quasi-Elliptic Cohomology and its Power Operations", arXiv:1612.00930.
- Landweber, "Elliptic Curves and Modular Forms in Algebraic Topology: proceedings of a conference held at the Institute for Advanced Study", Princeton, September 1986, Lecture Notes in Mathematics.
- Eugene Lerman, "Orbifolds as stacks", 2008. arXiv:0806.4160v1.
- Katz, Mazur, "Arithmetic moduli of elliptic curves", Annals of Mathematics Studies, vol. 108, 1985.
- Mandell, May, Schwede, Shipley, "Model categories of diagram spectra", Proc. London Math. Soc. 82(2001).
- May, "Equivariant homotopy and cohomology theory", CBMS Regional Conference Series in Mathematics, vol. 91, 1996.
- Rezk, "Quasi-elliptic cohomology", 2011.
- Schommer-Pries, "Central extensions of smooth 2-groups and a finite-dimensional string 2-group", Geom. Topol. 15 (2011).
- Schwede, "Global Homotopy Theory", global.pdf.
- Strickland, "Morava E-theory of symmetric groups", Topology 37 (1998), no. 4.